*1998 World Automation Congress, Anchorage, Alaska*

# CONSTELLATION TEMPLATES: AN APPROACH TO AUTONOMOUS FORMATION FLYING

**RANDAL W. BEARD**
*Brigham Young University*

**FRED Y. HADAEGH**
*Jet Propulsion Laboratory*
*California Institute of Technology*

## ABSTRACT

Multiple spacecraft flying in formation is a new paradigm that is key to the success of several future NASA missions. These missions include space-based interferometers, where a constellation of spacecraft coordinate their motion to create a large baseline interferometer that is easy to reconfigure and reorient. The coordination and control of a constellation of spacecrafts dictates several interesting design requirements, including efficient architectures and algorithms for formation acquisition, formation reorientation and formation resizing. This paper proposes the use of constellation templates as a uniform architecture to address these problems. A constellation template is a virtual pattern, with an inertial position and orientation, that specifies the desired position and attitude of each spacecraft within the constellation. Using constellation templates, efficient algorithms for formation acquisition and reorientation are developed.

**KEYWORDS:** formation Flying, interferometry, spacecraft constellation control.

## INTRODUCTION

NASA is currently planning several missions that will require multiple spacecraft flying in precise formations. These missions include space-based interferometers, where several spacecraft with precisely maintained baselines reflect light to a combiner spacecraft. The success of space-based interferometers, and other missions requiring formation flying, will depend on precise coordination and control of the spacecraft in the formation. Precisely maintaining the baseline of a space-based interferometer requires that a laser metrology system be used to measure the distance between the spacecraft. Since the metrology system is difficult to configure, it is desirable that the motions of the spacecraft are such that laser metrology system maintains sensor lock. This constrains the motion of the constellation to resemble a rigid body. In this paper we propose a simple architecture for addressing the tasks of formation acquisition and reorientation. The coordination and control of multiple spacecraft flying in formation was first addressed in [1].

# DEFINITIONS

The coordinate systems used in this paper are in Figure 1. The coordinate system labeled

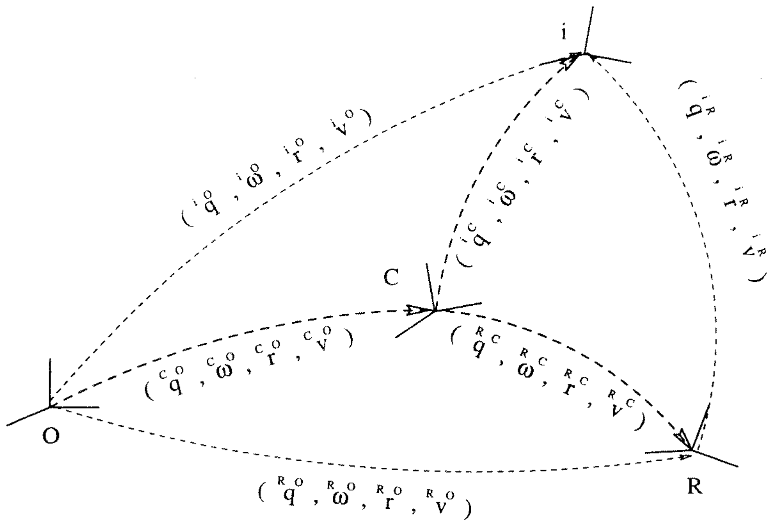


Figure 1: The geometry of formation flying.

**0** is the inertial coordinate system, **C** is the constellation coordinate system, **R** is a point of rotation for the constellation and will be used in formation reorientation, and **i** is the coordinate system associated with the $i^{\text{th}}$ spacecraft. The variables $(q,\ \omega,\ r,\ v)$ represent the attitude (unit) quaternion, the angular velocity, the position, and the translational velocity, respectively. The superscript after the variable denotes the reference frame in which the variable is measured, the superscript before the variable denotes the quantity being referenced. Hence ${}^{i}\mathbf{q}^{C}$ is the attitude of the $i^{\text{th}}$ spacecraft with respect to the coordinate frame C. A subscript $d$ will denote a desired quantity.

To simplify the expression for the kinematic constraints on the spacecraft it is convenient to write the attitude quaternion as $\mathbf{q} = \begin{pmatrix} \eta, & \epsilon \end{pmatrix}^T = \begin{pmatrix} \mathbf{e}^T \cos\left(\frac{\phi}{2}\right), & \sin\left(\frac{\phi}{2}\right) \end{pmatrix}^T$, where $\mathbf{e}$ is the Euler axis of rotation and $\phi$ is the angle of rotation about $\mathbf{e}$. We will assume that each spacecraft is governed by the following rigid body dynamic equations [1, 2]:

$$
\begin{aligned}
\frac{d\,{}^{i}\eta^{0}}{dt} &= \frac{1}{2}({}^{i}\epsilon^{0\,i}\omega^{O} - {}^{i}\omega^{O} \times {}^{i}\eta^{0}), & \frac{d\,{}^{i}\epsilon^{0}}{dt} &= -\frac{1}{2}({}^{i}\omega^{O} \cdot {}^{i}\eta^{0}), \\
\frac{d(I_i\,{}^{i}\omega^{O})}{dt} &= I_i \frac{d\,{}^{i}\omega^{O}}{dt} + {}^{i}\omega^{O} \times (I_i\,{}^{i}\omega^{O}) = \tau_c^i + \tau_e^i, & & \\
\frac{d\,{}^{i}\mathbf{r}^{O}}{dt} &= {}^{i}\mathbf{v}^{O}, & M_i \frac{d\,{}^{i}\mathbf{v}^{O}}{dt} &= f_c^i + f_e^i, \qquad (1)
\end{aligned}
$$

where $\tau_c^i$ is the control torque, $\tau_e^i$ is the environmental disturbance torque, $f_c^i$ is the control force and $f_e^i$ is the environmental disturbance force. The first three equations describe the rotational dynamics of the spacecraft, and the last two equations describe the translational dynamics.

A constellation template is defined as a virtual pattern, that specifies the desired relative position and orientation of each spacecraft with respect to the coordinate system **C**. Let $\tilde{\mathcal{M}} = \{\tilde{M}_i\}_{i=1}^N$, where $\tilde{M}_i$ is the modeled mass of the $i^{\text{th}}$ spacecraft, $\tilde{\mathcal{I}} = \{\tilde{I}_i\}_{i=1}^N$, where $\tilde{I}_i$ is the modeled inertia of the $i^{\text{th}}$ spacecraft, $\mathcal{Q}_d(t) = \{{}^{i}\mathbf{q}_d^C\}_{i=1}^N$, where ${}^{i}\mathbf{q}_d^C$ is the desired attitude quaternion of the $i^{\text{th}}$ spacecraft with respect to $\mathbf{C}$, and $\mathcal{R}_d(t) = \{{}^i\mathbf{r}_d^C\}_{i=1}^N$, where ${}^i\mathbf{r}_d^C$ is the desired position of the $i^{\text{th}}$ spacecraft with respect to $\mathbf{C}$. We assume that the desired *relative* velocity and angular velocity are zero for each spacecraft: the constellation as a whole can translate and rotate, but the relative position and orientation of each spacecraft in the constellation remains fixed.

A constellation template is defined as the following tuple

$$T(\mathbf{C}(t)) = \left\{\tilde{\mathcal{M}}, \tilde{\mathcal{I}}, \mathcal{Q}_d, \mathcal{R}_d, \right\}, \tag{2}$$

where the coordinate from $\mathbf{C}$ is allowed to have time varying position and orientation. The current state of the constellation $S$ can be described in notation similar to that of a template. Let

$$S(t) = \{\mathcal{M}, \mathcal{I}, \mathcal{Q}, \Omega, \mathcal{R}, \mathcal{V}\},$$

where $\mathcal{M} = \{M_i\}_{i=1}^N$ and $M_i$ is the actual mass of the $i^{\text{th}}$ spacecraft, $\mathcal{I} = \{I_i\}_{i=1}^N$ and $I_i$ is the actual inertia of the $i^{\text{th}}$ spacecraft, $\mathcal{Q}(t) = \{{}^i\mathbf{q}^O(t)\}_{i=1}^N$ and ${}^i\mathbf{q}^O$ is the attitude quaternion of the $i^{\text{th}}$ spacecraft with respect to $\mathbf{O}$, $\Omega(t) = \{{}^i\omega^O(t)\}_{i=1}^N$ and ${}^i\omega^O$ is the angular velocity of the $i^{\text{th}}$ spacecraft with respect to $\mathbf{O}$, $\mathcal{R}(t) = \{{}^i\mathbf{r}^O(t)\}_{i=1}^N$ and ${}^i\mathbf{r}^O$ is the position of the $i^{\text{th}}$ spacecraft with respect to $\mathbf{O}$, and $\mathcal{V}(t) = \{{}^i\mathbf{v}^O(t)\}_{i=1}^N$ and ${}^i\mathbf{v}^O$ is the velocity of the $i^{\text{th}}$ spacecraft with respect to $\mathbf{O}$.

Given a constellation $S$, we can define its norm with respect to the template $T(\mathbf{C})$ as

$$\|S(t)\|_T(\mathbf{C}(t)) = \sum_{i=1}^N \left( \left\|{}^i\mathbf{r}_d^C - {}^i\mathbf{r}^C(t)\right\|^2 + \left\|{}^i\mathbf{v}^C(t)\right\|^2 + \left\|{}^i\mathbf{q}_d^C - {}^i\mathbf{q}^C(t)\right\|^2 + \left\|{}^i\omega^C(t)\right\|^2 \right),$$

where ${}^i\mathbf{r}^C(t) = {}^i\mathbf{r}^O(t) - {}^C\mathbf{r}^O(t)$, ${}^i\mathbf{v}^C(t) = {}^i\mathbf{v}^O(t) - {}^C\mathbf{v}^O(t) - {}^C\omega^O(t) \times {}^i\mathbf{r}^C(t)$, ${}^i\mathbf{q}^C(t) = \left({}^C\mathbf{q}^O(t)\right)^* {}^i\mathbf{q}^O(t)$ and ${}^i\omega^C(t) = {}^i\omega^O(t) - {}^C\omega^O(t)$.

In the next two sections we will show how these definitions can be used to state and solve the problems of formation acquisition and formation reorientation.

## FORMATION ACQUISITION

The formation acquisition problem is to control each spacecraft such that the constellation $S$ is in the formation specified by the template $T$. The problem can be broken into two steps. First, find the orientation of the template that best matches the current configuration. Second, avoiding collisions, control each spacecraft such that its coordinate axis aligns with the appropriate coordinate axis in the template. These two steps can be written formally as

**Step 1:** Find $\mathbf{C}_o$ such that $\mathbf{C}_o = \arg\min \|S(t_0)\|_{T(\mathbf{C})}$,

**Step 2:** Design a control law for each spacecraft such that $\|S(t)\|_{T(\mathbf{C}_0)} \to 0$ as $t \to \infty$, subject to the constraint that $\left\|{}^j\mathbf{r}^O - {}^i\mathbf{r}^O\right\| > \epsilon$, $\forall i \neq j$.

The first step finds the position and orientation of the template that best matches the current constellation of the spacecraft. The second step regulates the spacecraft to the positions and orientations specified by the template, subject to the constraint that the spacecraft remain a distance $\epsilon$ apart from each other. We will describe an approach to each of these steps.

## Step 1:

A simple approach to step 1 is to ignore the desired orientation of the spacecraft and to find a template that minimizes the distance from each spacecraft to its respective desired position in the constellation. If

$$F = \sum_{i=1}^{N} \left\| {}^i\mathbf{r}_d^C - {}^i\mathbf{r}^C \right\|^2 = \sum_{i=1}^{N} \left\| \left({}^C\mathbf{q}^O\right) \left({}^i\mathbf{r}_d^C\right) \left({}^C\mathbf{q}^O\right)^* - {}^i\mathbf{r}^O + {}^C\mathbf{r}^O \right\|^2,$$

then a simple gradient descent algorithm that minimizes $F$ is given below.

**Given:** ${}^i\mathbf{r}^O$, ${}^i\mathbf{r}_d^C$, $i = 1 \ldots N$.

**Initialize:**

$$\begin{aligned} {}^C\mathbf{q}^O(0) &= \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}^T \\ {}^C\mathbf{r}^O(0) &= \frac{1}{N}\sum_{i=1}^{N} {}^i\mathbf{r}^O - \frac{1}{N}\sum_{i=1}^{N} \left({}^C\mathbf{q}^O(0)\right) \left({}^i\mathbf{r}_d^C\right) \left({}^C\mathbf{q}^O(0)\right)^*. \end{aligned}$$

**Iterative Step:**

$$\begin{aligned} {}^C\mathbf{q}^O(k+1) &= {}^C\mathbf{q}^O(k) - \gamma \frac{\partial F}{\partial {}^C\mathbf{q}^O}\left({}^C\mathbf{q}^O(k), {}^C\mathbf{r}^O(k)\right) \\ {}^C\mathbf{r}^O(k+1) &= \frac{1}{N}\sum_{i=1}^{N} {}^i\mathbf{r}^O - \frac{1}{N}\sum_{i=1}^{N} \left({}^C\mathbf{q}^O(k+1)\right) \left({}^i\mathbf{r}_d^C\right) \left({}^C\mathbf{q}^O(k+1)\right)^*. \end{aligned}$$

In simulation, we have observed that for $\gamma$ small enough, this algorithm converges to the appropriate values for ${}^C\mathbf{r}^O$ and ${}^C\mathbf{q}^O$. Various practical and theoretical questions still need to be investigated. For example, convergence and rate of convergence of the algorithm. Also, we need to show that the Hessian of $F$ is positive definite and investigate the existence of local minima of $F$ for large $N$. In addition, there are more efficient methods to do gradient search, and the possible gains in performance need to studied.

## Step 2:

The second step in formation acquisition is to control each spacecraft to their desired positions in the constellation, avoiding collisions. Our approach to this problem will be to define a positive definite, radially unbounded, pseudo-energy function such that energy will be zero when the spacecraft are in the desired configuration and the energy function goes to infinity as spacecraft the collide. Let

$$P_i = \frac{1}{2}\left\| {}^i\mathbf{r}_d^C - {}^i\mathbf{r}^C \right\|^2 + \sum_{j \neq i} \left( \sigma \left( \frac{D}{\left\| {}^j\mathbf{r}^C - {}^i\mathbf{r}^C \right\| - 1} \right) \right)^2,$$

where $\sigma(x) = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$. Then $P = \sum_{i=1}^{N} \left( \alpha P_i({}^i\mathbf{r}^C) + \frac{1}{2}\left\| {}^i\mathbf{v}^C \right\|^2 \right)$ satisfies these conditions and

$$u_i = -\alpha \frac{\partial P_i}{\partial {}^i\mathbf{r}^C} - \beta {}^i\mathbf{v}^C \tag{3}$$

guarantees that $\dot{P}$ is negative semi-definite. Additional heuristic terms must be added to $u_i$ to guarantee that $P$ converges to zero.

## FORMATION REORIENTATION

The formation reorientation problem is to re-target the entire constellation of spacecrafts under the constraint that the formation must move as a rigid body. The problem can be formally stated as follows. Given (1) the constellation $S$, (2) the template $T(\mathbf{C})$ such that $\|S(0)\|_{T(\mathbf{C}(0))} < \delta$, (3) a point of rotation ${}^R\mathbf{r}^O$, and (4) a desired orientation of the template ${}^C\mathbf{q}_d^O$, determine a control law, for each spacecraft, such that $\left\|{}^C\mathbf{q}_d^O - {}^C\mathbf{q}^O(t)\right\| \to 0$, subject to the constraint that $\|S(t)\|_{T(\mathbf{C}(t))} < \epsilon, \forall t > 0$. The problem can be decomposed into three steps.

**Step 1:** First, the template is treated as a rigid body, and a control law is derived for controlling the orientation of the *template*.

**Step 2:** Next, the motion of the template is used to define the desired position and attitude trajectories for each spacecraft.

**Step 3:** Finally, control laws are designed for each spacecraft that cause them to track these desired trajectories.

### Step 1.

The constellation template $T$ can be used to define a virtual rigid body. The inertia of this rigid body about ${}^R\mathbf{r}^O$ is [3, pp. 43]

$$J = \sum_{i=1}^{N} M_i \left[\left\|{}^i\mathbf{r}_d^R\right\|^2 I - ({}^i\mathbf{r}_d^R)({}^i\mathbf{r}_d^R)^T\right],$$

where $I$ is the the $3 \times 3$ identity matrix. The rotational dynamics of this virtual rigid body about ${}^R\mathbf{r}^O$ are given by [1, 2]:

$$\begin{aligned}
\frac{d\,{}^R\eta^0}{dt} &= \frac{1}{2}({}^R\epsilon^0\,{}^R\omega^O - {}^R\omega^O \times {}^R\eta^0) \\
\frac{d\,{}^R\epsilon^0}{dt} &= -\frac{1}{2}({}^R\omega^O \cdot {}^R\eta^0) \\
\frac{d(J\,{}^R\omega^O)}{dt} &= J\frac{d\,{}^R\omega^O}{dt} + {}^R\omega^O \times (J\,{}^R\omega^O) = \tau_R.
\end{aligned} \tag{4}$$

A virtual control torque $\tau_R$ that asymptotically stabilizes the template is given by [1, 4]

$$\tau_R = K_1\left[{}^C\epsilon_d^0({}^C\eta_d^0 - {}^R\eta^0) - ({}^C\epsilon_d^0 - {}^R\epsilon^0)\,{}^C\eta_d^0 + {}^C\eta_d^0 \times {}^R\eta^0\right] - K_2 J\,{}^R\omega^O, \tag{5}$$

where $K_1$ and $K_2$ are positive constants and where we have assumed that the attitude of $R$ is identical to the attitude of $C$, i.e., ${}^R\mathbf{q}_d^O = {}^C\mathbf{q}_d^O$.

Given the control law in equation (5), the virtual dynamics of the constellation can be integrated to produce desired trajectories for the variables ${}^C\mathbf{q}^O(t) = \begin{pmatrix} {}^C\eta^0(t) & {}^C\epsilon^0(t) \end{pmatrix}^T$, and ${}^C\omega^O(t)$. The next step is to translate these trajectories into desired trajectories for each spacecraft to track.

**Step 2.**

Geometrical arguments can be used to show that the desired orientation, angular velocity, position and velocity satisfy the following equations:

$$
\begin{aligned}
&{}^i\mathbf{q}_d^C(t) = \left({}^R\mathbf{q}^O(t)\right)^* \left({}^i\mathbf{q}_d^C(t_0)\right) \left({}^R\mathbf{q}^O(t)\right), \qquad {}^i\omega_d^C(t) = -{}^R\omega^O(t), \\
&{}^i\mathbf{r}_d^C(t) = \left({}^R\mathbf{q}^O(t)\right)^* \left({}^i\mathbf{r}_d^C(0) - {}^R\mathbf{r}^C\right) \left({}^R\mathbf{q}^O(t)\right) + {}^R\mathbf{r}_d^C, \; {}^i\mathbf{v}_d^C(t) = {}^R\omega^O(t) \times \left({}^i\mathbf{r}_d^C - {}^R\mathbf{r}^C\right), \\
&{}^i\mathbf{a}_d^C(t) = {}^R\omega^O(t) \times {}^i\mathbf{v}_d^C(t) + \frac{d{}^R\omega^O}{dt}(t) \times \left({}^i\mathbf{r}_d^C - {}^R\mathbf{r}^C\right).
\end{aligned}
$$

**Step 3.**

The third step is to design tracking controllers that cause each spacecraft to track the desired trajectories. A control law for attitude tracking control is derived in [1, 4] and is given by

$$
\begin{aligned}
\tau_{ci} = K_{i1} &\left[{}^i\epsilon_d^C \left({}^i\eta_d^C - {}^i\eta^C\right) - \left({}^i\epsilon_d^C - {}^i\epsilon^C\right) - {}^i\eta_d^C \times {}^i\eta^C\right] \\
&- \frac{1}{2}{}^i\omega_d^C \times \left(\tilde{I}_i \left({}^i\omega_d^C - {}^i\omega^C\right)\right) + K_{2i}\tilde{I}_i \left({}^i\omega_d^C - {}^i\omega^C\right).
\end{aligned}
$$

The translational motion is controlled via

$$
f_{ci} = \tilde{M}_i \left[{}^i\mathbf{a}_d^C + K_{vi}\left({}^i\mathbf{v}_d^C - {}^i\mathbf{v}^C\right) + K_{pi}\left({}^i\mathbf{r}_d^C - {}^i\mathbf{r}^C\right)\right],
$$

where $K_{vi}$ and $K_{pi}$ are positive constants. This control law, together with the translational dynamics of the spacecraft given in equation (1) result in second order error dynamics given by $\ddot{e} + K_{vi}\dot{e} + K_{pi}e = f_e$, where $e = {}^i\mathbf{r}^C - {}^i\mathbf{r}_d^C$.

A critical question that needs to be addressed is how to choose the control gains such that $\|S(0)\|_T < \delta$ implies that $\|S(t)\|_T < \epsilon$ for all $t > 0$.

# CONCLUSIONS

In this paper we have outlined an approach to autonomous formation flying. The central theme is the use of constellation templates. Templates allow the tasks of formation acquisition and reorientation to be addressed in a uniform and consistent way. Simple algorithms for formation acquisition and reorientation were also derived.

# ACKNOWLEDGMENT

This work was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

# References

[1] P. K. C. Wang and F. Y. Hadaegh, "Coordination and control of multiple microspacecraft moving in formation," *The Journal of the Astronautical Sciences*, vol. 44, no. 3, pp. 315–355, 1996.

[2] J. R. Wertz, ed., *Spacecraft Attitude Determination and Control*. Kluwer Academic Publishers, 1978.

[3] P. C. Hughes, *Spacecraft Attitude Dynamics*. John Wiley & Sons, 1986.

[4] J. T.-Y. Wen and K. Kreutz-Delgado, "The attitude control problem," *IEEE Transactions on Automatic Control*, vol. 36, pp. 1148–1162, October 1991.